Math 252, Advanced Calculus II
First Midterm Exam

March 19, 2019

Name: _____

Number: _____

Question:	1	2	3	4	5	Total
Grade:						
Out of:	22	18	18	22	20	100

Question 1. Consider the following equations

$$\begin{cases} F: \ 4x - 5y + z^3 - 3u + v^2 = 0 \\ G: \ x^2 + 2y^2 + z + u^2 - 2v + 3w = 6 \\ H: \ 2x^3 + 4z^2 + w - u^2 + v^2 = 9 \end{cases}$$

a) Show that near the point $p_0 = (x, y, z, u, v, w) = (2, 0, 0, 0, 1, -1)$, we can solve u, v, w uniquely as functions of x, y, z .
b)Compute $\frac{\partial u}{\partial x}$ at the point p_0. (Hint: Use Implicit function Theorem).

Question 2. Write the integral $\int_0^2 \int_0^2 f(\sqrt{x^2 + y^2})\ dx\ dy$ in polar coordinates.

Question 3. Using transformation $u = xy$ and $v = xy^3$, find the area of the region R in the first quadrant bounded by the curves $xy = 2,\ xy = 4,\ xy^3 = 3$ and $xy^3 = 6$.

Question 4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3;\ T(x, y, z) = (u, v, w)$ with

$$\begin{cases} u(x, y, z) = xyz + c\ y^2 \\ v(x, y, z) = yz^2 - 3x \\ w(x, y, z) = 4xy + 6y + z^3 \end{cases}$$

a) Find $DT(1, 1, 1)$.
b) For which values of c can the system of equations be solved for x, y, z as functions of u, v, w near $(1, 1, 1)$?
b) Find $\frac{\partial y}{\partial v}$ at the point $(1, 1, 1)$.

Question 5. What is the work done $\int_C \vec{F}.d\vec{r}$ by moving in the force field

$$F(x, y) = (2x^3 + 1)\vec{i} + (4\pi y^3 \sin(\pi y^4))\vec{j}$$

along the curve $y = x^4$ from $(-1, 1)$ to $(1, 1)$.