

**Math 252, Advanced Calculus II**  
**Second Midterm Exam**

April 09, 2019

Question:	1	2	3	4	5	Total
Grade:						
Out of:	20	20	20	20	20	100

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(1) a) Evaluate the integral  $\int_0^1 \int_{e^x}^e \frac{y}{\ln y} dy dx$ .

b) Rewrite the integral  $\int_{-2}^1 \int_{y^2-1}^{1-y} f(x, y) dx dy$  with the order  $dy dx$ .

(2) Use a suitable transformation (substitution) to evaluate

$$\int_R \int \frac{x^2 y}{1 + xy} dA$$

where  $R$  is the region bounded by  $y = \frac{1}{x^2}$ ,  $y = \frac{2}{x^2}$ ,  $y = \frac{1}{2x}$ ,  $y = \frac{1}{x}$ .

(3) a) Find the value of the constant  $\mathbf{k}$  for which the vector field  $\vec{F}(x, y) = (kx^3y^2 - 2xy^3)\vec{i} + (2x^4y - 3x^2y^2 + 4y^3)\vec{j}$  is conservative.

b) For the value of  $\mathbf{k}$  found above, evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is the union of the line segments from  $(-5, 0)$  to  $(3, 7)$  and  $(3, 7)$  to  $(2, 2)$ .

(4) Use Green's theorem to evaluate  $\oint_C \ln(2\pi + \arctan x) dx + (2x + \frac{1}{1+y^2}) dy$ ,

where  $C$  is the circle  $(x+2)^2 + (y-2)^2 = 4$  traversed in counterclockwise direction.

(5) Use Stoke's Theorem, calculate

$$\oint_C (5y - 2z) dx + (3x - z) dy + (2y - x) dz$$

where  $C$  is the triangle with vertices  $A(3, 0, 0)$ ,  $B(0, 1, 0)$ ,  $C(0, 0, 2)$ .